and the turbulent Schoenherr law for no drag reduction $C_{F,0}$,

$$\log R_L C_{F,0} = 0.242 C_{F,0}^{-1/2} \tag{10}$$

The percentage of drag reduction, $\%D.R.$, is also plotted in Fig. 2

$$\%D.R. = (1 - C_F/C_{F,0})100\% \tag{11}$$

where C_F is the drag coefficient for polymer solutions and $C_{F,0}$ is the drag coefficient for no drag reduction at the same Reynolds number. The results are most favorable.

It should be noted that the maximum drag reduction is predicted on the basis of a theoretical model. In practice high-shear stresses will probably mechanically degrade the polymer molecules and diminish the drag reduction so that the maximum drag reduction may not be attained. The actual friction line lies then between the Schoenherr line of no drag reduction and the line of maximum reduction. This line may be determined by the method of Ref. 2.

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Added Mass of a Circular Cylinder in Contact with a Rigid Boundary

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Introduction

THE calculation of the forces exerted on an object placed in an accelerating flowfield is a fluid mechanics problem of considerable practical importance. For example, in the calculation of wave forces acting on submerged bodies it is necessary, in general, to know both the added mass and drag coefficient for the particular shape in question. The wave force is generally considered to consist of two parts, drag and inertia. However, if the object involved is large in comparison to the amplitude of the fluid motion, separation does not occur and the resulting wave induced oscillatory flow may be considered an unseparated, potential flow about a fixed object. In this case the drag coefficient is necessarily zero while the added mass coefficient is dependent on the shape of the body in question and its proximity to any rigid boundaries. In problems of this type, the potential flow value of the added mass coefficient is of considerable practical importance since it is the basic shape dependent factor required for calculating forces.

One configuration of particular importance in view of its prevalence in engineering structures is the circular cylinder. When a circular cylinder is immersed in a fluid of infinite extent its added mass coefficient is easily calculated and has the well-known value of $C_m = 1.0$. (The inertia coefficient is defined as $C_I = 1.0 + C_m$ and accordingly has the value of 2.0 for the circular cylinder.) However, when the circular cylinder is placed near a solid boundary, as shown in Fig. 1, and the fluid accelerated parallel to the boundary, the added mass is increased somewhat due to the presence of the rigid boundary. This particular configuration consisting of a circular cylinder in contact with a rigid boundary has obvious practical significance in application to wave forces exerted on bottom mounted structures such as submerged pipelines and oil storage tanks. In these cases it is of interest to know the effect of the proximity of the bottom on the added mass coefficient.

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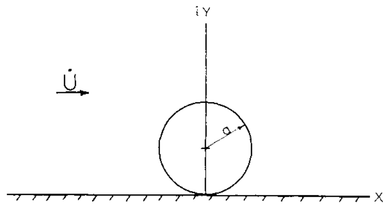

Fig. 1 Definition sketch.

The problem under consideration was considered previously by Dalton and Helfinstine[1] as the special limiting case of two circular cylinders approaching touching. Their approximate method, based on the use of images, involved repeated application of Milne-Thomson's circle theorem in order to generate potential flow past two cylinders in close proximity. Half of this configuration corresponds to the present situation of a cylinder touching a plane boundary in view of the symmetry involved. However, as touching became imminent their solution broke down locally on account of the singularity at the point of contact and, therefore, the accuracy of the value of C_m so obtained is questionable. This is unfortunate since the just touching case is the most important with respect to practical application.

In the present Note the case of a circular cylinder in contact with a plane boundary is treated. The complex potential for this case is expressed in closed form and is applied to calculate the net force in the direction of fluid acceleration. From the inertia coefficient so obtained, the added mass coefficient for the circular cylinder in contact with a plane boundary is calculated in closed form without recourse to approximate methods.

Inertial Force

It is well known that the drag force exerted on an object in a potential flow (without circulation) is zero while the added mass or inertial force is dependent upon the shape of the object and proximity to solid boundaries. In order to evaluate this inertia force it is necessary to determine the pressure distribution around the cylinder. Such information can be obtained from consideration of the unsteady form of Bernoulli's equation for an incompressible fluid,

$$P/\rho + q^2/2 + gh + \partial\phi/\partial t = \Pi(t) \tag{1}$$

where P denotes the pressure at any general point within the fluid, q the local velocity, h the elevation above some arbitrary datum, ρ the fluid density, g the acceleration of gravity, ϕ the velocity potential and $\Pi(t)$ some function of time only.

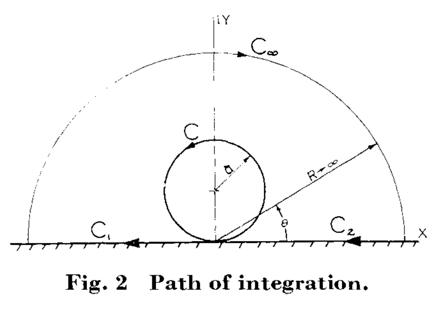

Fig. 2 Path of integration.

The net force acting on the cylinder may be obtained by solving Eq. (1) for the pressure and integrating around the surface of the cylinder. However, since the cylinder represents a closed curve, the $\Pi(t)$ term will contribute nothing to this integration and the hydrostatic pressure term gh will contribute only a constant buoyant force. Moreover, the $q^2/2$ term is symmetrical about the y-axis and, consequently, will yield no net contribution to the x-component of force. We are left, therefore, with

$$-\rho\partial\phi/\partial t \tag{2}$$

as the only term in the expression for pressure which will contribute to a horizontal component of force. Thus, the x and y components of inertial force exerted on the circular cylinder may be written as

$$X = -\oint_c Pdy = \rho\partial/\partial t \oint_c \phi dy \tag{3}$$

and

$$Y = \oint_c Pdx = -\rho\partial/\partial t \oint_c \phi dx \tag{4}$$

respectively. (It may be noted that the $q^2/2$ term is not symmetrical with respect to the x-axis and, therefore, a lift force proportional to velocity squared does exist.)

Combining Eqs. (3) and (4)

$$-Y + iX = \rho\partial/\partial t \oint_c \phi dz \tag{5}$$

where the complex number z is defined as $z = x + iy$. Since the stream-function is constant (or zero) on the surface of the cylinder we may write Eq. (5) as

$$-Y + iX = \rho\partial/\partial t \oint_c w(z)dz \tag{6}$$

where the complex potential is defined as $w(z) = \phi + i\psi$ and ϕ and ψ denote the velocity potential and stream function, respectively.

The complex potential for flow past a circular cylinder touching a plane boundary as depicted in Fig. 1 is given by Milne-Thomson[2] as

$$w(z) = a\pi U \coth(a\pi/z) \tag{7}$$

where a denotes the radius of the cylinder and U the freestream velocity. Using this expression, Eq. (6) becomes

$$-Y + iX = \pi a\rho\dot{U} \oint_c \coth(a\pi/z)dz \tag{8}$$

where the integration is to be carried out around the closed curve C representing the surface of the cylinder. However, since $w(z)$ is an analytic function, we may apply Cauchy's integral theorem to the region bounded by the closed curve C representing the cylinder, the x-axis and the arc of radius R as labeled C_∞ in Fig. 2. Accordingly, the integral represented in Eq. (8) may be expressed as the sum of the integrals along the various segments of the complete contour as

$$\int_c = \int_{c_1} + \int_{c_2} + \int_{c_\infty} \tag{9}$$

It may be noted that $\cosh(\pi a/z)$ is analytic throughout the region bounded by the lines C, C_1, C_2 and C_∞. However, since $\coth(\pi a/x)$ is an odd function about $x = 0$ the integrals along C_1 and C_2 cancel each other leaving

$$-Y + iX = \pi\rho a\dot{U} \oint_{c_\infty} \coth(\pi a/z)dz \tag{10}$$

where the integration is to be carried out along the semicircular arc labeled C_∞ in Fig. 2.

In order to evaluate the integral occurring in Eq. (10) we write $z = Re^{i\theta}$ and expand the integrand for large values of R. This gives,

$$\coth(a\pi/z) = (R/a\pi)[1 + \tfrac{1}{3}(a\pi/R)^2e^{-i2\theta} + O(1/R^4)]e^{i\theta}, R\to\infty \tag{11}$$

Substituting Eq. (11) into Eq. (10) and carrying out the integration gives the x and y components of inertial force as

$$Y = 0 \tag{12}$$

$$X = \rho\pi a^2\dot{U}(\pi^2/3) \tag{13}$$

where the inertial force is defined as positive in the positive x-direction, i.e., in the direction of the acceleration of the ambient fluid. According to the usual definitions of added mass and inertial coefficients, we have

$$C_I \equiv X/\pi a^2\rho\dot{U} \equiv 1.0 + C_m \tag{14}$$

Accordingly, the added mass coefficient is given as

$$C_m = \pi^2/3 - 1.0 = 2.29 \tag{15}$$

It may be noted that Dalton and Helfinstine's value of $C_m = 2.22$ is only slightly in error when compared to the exact value as given in Eq. (15).

Conclusions

The exact solution for the added mass coefficient for a circular cylinder in contact with a plane boundary has been developed. The value of $C_m = 2.29$ so obtained is shown to be considerably larger than the value of $C_m = 1.0$ for a circular cylinder in an infinite fluid. The proximity of the rigid boundary has the effect of increasing the value of the added mass coefficient. The results are of particular interest in the calculation of wave forces on bottom mounted structures such as pipelines and submerged tanks.

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Mean Velocity Profiles for Turbulent Shear Flow

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In any investigation of either turbulent boundary-layer flow or enclosed turbulent flow, it is necessary to utilize some known expression for the mean velocity profile. One manner by which such an expression is commonly obtained is through the use of a three-layer scheme which divides the flowfield into three regions—a wall region, a buffer zone, and a turbulent core. The mean velocity profile expressions utilized with this model are given by Schlichting[1] to be

$$\bar{U}/u^* = yu^*/\nu \quad \text{for} \quad yu^*/\nu \le 5 \tag{1a}$$

$$\bar{U}/u^* = 5.0 \ln(yu^*/\nu) - 3.05 \quad \text{for} \quad 5 \le yu^*/\nu \le 30 \tag{1b}$$

$$\bar{U}/u^* = 2.5 \ln(yu^*/\nu) + 5.5 \quad \text{for} \quad yu^*/\nu \ge 30 \tag{1c}$$

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